Topic 4

Business Mathematics

Session Objectives

- Explain and calculate an expected value
- Demonstrate the use of expected values in simple decision making situations
- Explain the limitations of the expected value technique
- Calculate a correlation coefficient and a coefficient of determination
- Establish a linear function using regression analysis and interpret the results

Expected Values

- It is the sum of the probability of each possible outcome of the experiment multiplied by the outcome value.
- Probabilities are based on an analysis of past data.
- Can be calculated as the weighted average of a probability distribution.

Expected Values (EV)

Expected value = Σpx

Where:

Σ = sum of

p= probability of outcome occurring

x = outcome

Example:

If the probability of winning \$x is p then the expectation (or EV) is p x \$x

Question

- An experiment consists of tossing a fair coin three times. Let X denote the number of heads which appear. Then the possible values of X are 0, 1, 2 and 3. The corresponding probabilities are 1/8, 3/8, 3/8, and 1/8. What is the expected value of X?

Solution

$$0\left(\frac{1}{8}\right)+1\left(\frac{3}{8}\right)+2\left(\frac{3}{8}\right)+3\left(\frac{1}{8}\right)=\frac{3}{2}\,.$$

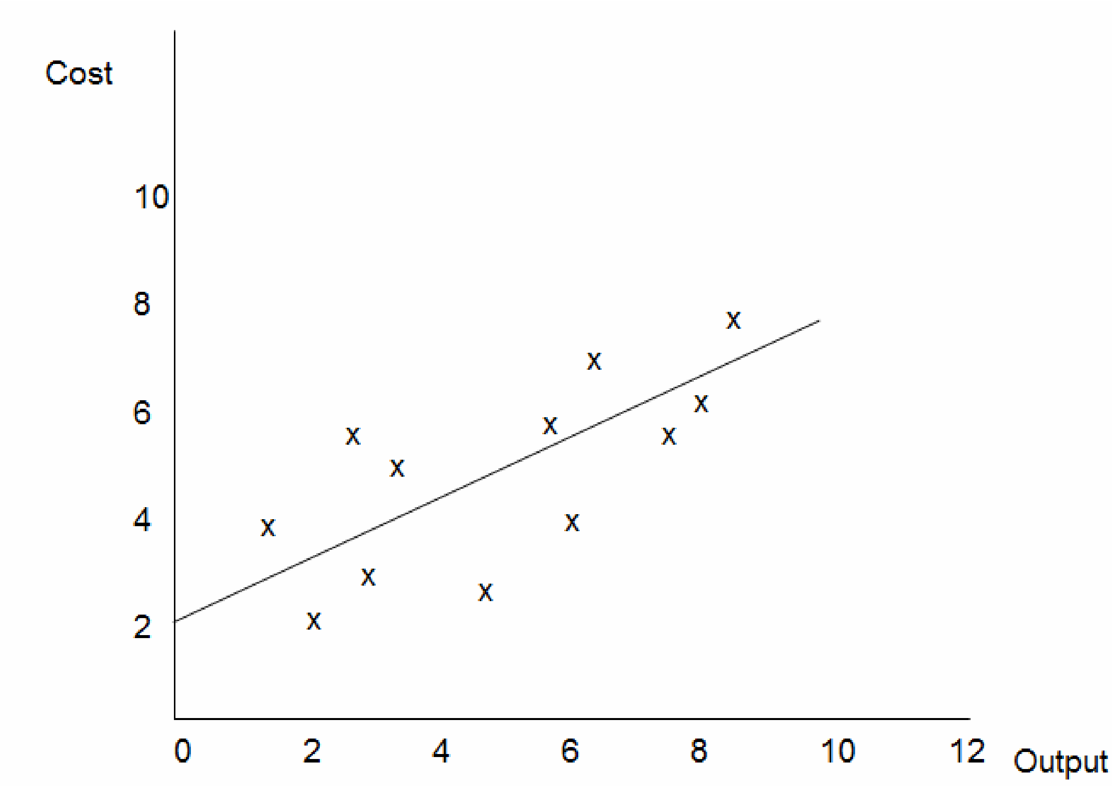
Acceptance criteria of Expected Value

- Accept projects only if EV is positive. While deciding between projects accept a project that has the highest EV.

Limitations of Expected Value Technique

- Probabilities are based on past data and may not necessarily hold true in the future
- The technique ignores time value of money
- Expected values are long term average values and may not be suitable for making one-off decisions

Linear Regression Analysis



The line of best fit is the cost equation y= a + bx where a = fixed cost and b = variable cost per unit

Establishing a Linear Function

$$y = a + bx$$

Where, $a = \bar{y} - b\bar{x}$

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Where, n = number of pairs of x,y values

Example

- The following table shows the units of good produced and the total costs incurred. Calculate the regression line of y on x

Unit Produced	Cost
1	400
2	450
3	500
4	650
5	700
6	700
7	800

Step 1: Tabulate the data

x	y
1	400
2	450
3	500
4	650
5	700
6	700
7	800

Step 2

x	y	xy	x^2
1	400	400	1
2	450	900	4
3	500	1500	9
4	650	2600	16
5	700	3500	25
6	700	4200	36
7	800	5600	49
28	4200	18700	140

Step 3: Value of b

$b = (n\Sigma xy - \Sigma x \Sigma y) / (n\Sigma x^2 - (\Sigma x)^2)$

$= (7 * 18700 - 28 * 4200) / (7 * 140 - 28^2)$

$= 67.85$

Step 4: Value of a

$a = (\Sigma y / n) - b(\Sigma x / n)$

$= (4200 / 7) - 67.85(28 / 7)$

$= 600 - 271.40$

$= 328.60$

Line of Best Fit

The Estimated line of best fit is:

$$Y = 328.60 + 67.85x$$

Correlation

- Correlation measures the extent of the relationship between two variables.
- Regression does not do that.

Correlation

- Is positive when the values of variables increase together
- Is negative when one variable increases and the other decreases
- When correlation is strong the estimated line of best fit is more reliable.
- If correlation is weak, the line of best fit calculated by linear regression might be insufficiently reliable.

Correlation Coefficient

- Correlation coefficient measures the intensity of the correlation
- Correlation coefficient (r) or Pearson's correlation coefficient =

$$r = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{(n\,\Sigma x^2 - (\Sigma x)^2)(n\,\Sigma y^2 - (\Sigma y)^2)}}$$

Example

Calculate the correlation coefficient for the regression line:

$$y = 32.84 + 6.79x$$

x	y	xy	x^2
1	40	40	1
2	45	90	4
3	50	150	9
4	65	260	16
5	70	350	25
6	70	420	36
7	80	560	49
28	420	1870	140

$$r = \frac{n \Sigma x y - \Sigma x \Sigma y}{\sqrt{(n \Sigma x^2 - (\Sigma x)^2)(n \Sigma y^2 - (\Sigma y)^2)}}$$

r = [7 (1870) – 28 (420)] / $\sqrt{7 (140)-28(28)] [7(26550) – (420) (420)}$

= 0.98

Interpretation of coefficient of correlation

- Value of r varies between +1 and -1
  - R = +1 means perfect positive linear correlation
  - R = -1 means perfect negative linear correlation
  - R = 0 means no correlation

Spurious Correlation

- This is also known as nonsense correlation.
- Two variables when compared show a high degree of correlation but may still have no direct connection.
  - Example: Number of television licenses and number of admissions to mental hospitals

Coefficient of Determination

- Square of coefficient of correlation.
- It is denoted by r^2 .
- It measures how much of the variation of the dependent variable is explained by the variation of independent variable.

Example

- r had a value of 0.98 so $r^2 = 0.96$ or 96%.
- Thus 96% of the variations in total cost are explained by the variations in activity level.

Sample Exam Question

- What is the approximate value of ‘a’ in the regression line denoted by $y = a + bx$, given

  $\Sigma x = 264$, $\Sigma y = 198$, $\Sigma x^2 = 10{,}792$, $\Sigma y^2 = 6220$, $\Sigma xy = 8{,}080$, $b = 0.415$ and $n = 11$?

  A. 2.63
  B. 8.04
  C. 9.96
  D. 18.02

Answer

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$$

$$= \frac{198}{11} - \frac{0.415\ (264)}{11}$$

$$= 8.04$$

Sample Exam Question

- Correlation coefficient can take all of the following values EXCEPT:
  - A. +1.6
  - B. +0.2
  - C. 0
  - D. –0.3

Answer

- **A**

- Coefficient of correlation ‘r’ can take any value between +1 and -1.