Topic 10

Limiting Factors

Session Objectives

- Identify a single limiting factor
- Determine the optimal production plan where an organisation is restricted by a single limiting factor
- Formulate a linear programming problem involving two variables
- Determine the optimal solution to a linear programming problem using a graphical approach
- Use simultaneous equations, where appropriate, in the solution of a linear programming problem

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Managing Limiting Factors

- A factor that restricts output.
- Objective should be to concentrate on those products/services that yield the largest contribution per limiting factor.
- Limiting factor decisions involve determination of the contribution earned by each different product from each unit of the limiting factor.

Managing Single Limiting Factor

- **Step 1:** Identify the limiting factor
- **Step 2:** Calculate contribution per unit for each product
- **Step 3:** Calculate contribution per unit of limiting factor

  Contribution per unit of limiting factor

  = Contribution per unit / Units of limiting factor required per unit

Managing Single Limiting Factor ....Cont'd

- **Step 4:** Rank products
- **Step 5:** Make products in rank order until scarce resources are used up

Example

Components	X	Y	Z
Contribution per unit	£12	£10	£6
Machine hours per unit	6	2	1
Estimated sales demand (units)	2,000	2,000	2,000

Capacity for the period is restricted to 12 000 machine hours. What should be the product mix?

Solution

- **Step 1:** Identify the limiting factor
  - *Machine hours*
- **Step 2:** Calculate contribution per unit for each product
  - *Given as X £12, Y £10, Z £6*

- **Step 3:** Calculate contribution per unit of limiting factor

  Contribution per unit of limiting factor

  = Contribution per unit / Units of limiting factor required per unit

  Thus, X = £12 / 6 = £2, Y= £10 / 2 = £5, Z=£6 / 1 = £6

- **Step 4:** Rank products (make product with highest contribution per unit of limiting factor first)
  - *Ranking: Z, Y, X*
- **Step 5:** Make products in rank order until scarce resources are used up
  - First 2000 units of Z should be made, then 2000 units of Y and finally with left machine hours X

Production	Machine Hours Used	Balance Machine Hours Available
2 000 units of Z	2000	10,000
2 000 units of Y	4000	6,000
1000 units of X	6,000	-

Managing Multiple Limiting Factor

- Linear programming is an operations research technique
- It can be used to decide on the optimum use of scarce resources.

Necessary Condition for Linear Programming

- There must be a linear relationship between the factors.

Stages in Linear Programming

- **Step 1:** Define the unknowns
- **Step 2:** Frame the constraints equation
- **Step 3:** State the objective functions
- **Step 4:** Graph the constraints
- **Step 5:** Graph objective function
- **Step 6:** Establishing feasible values for the unknowns

Example

Beans & Greens plc makes 2 products X and Y and has a shortage of labour which is limited to 10000 hours per annum. It wishes to maximise profits. Information on X and Y is as follows:

	Product X	Product Y
Labour hours per unit	2.5	5
Selling price per unit	£40	£50
Variable cost per unit	£25	£25
Contribution per unit	£15	£25

There is an unlimited demand for Product Y but demand for Product X is limited to 3000 units. How many units of X and Y should the company manufacture to maximise contribution?

Solution: Step 1

Let:

- x = number of units of X produced and sold during the year
- y= number of units of Y produced and sold during the year

Step 2: Formulate the constraints

Constraint	
Labour hours	$2.5x + 5y \leq 10000$
Maximum Sales	$x \leq 3000$
Cannot make a negative amount	$x \geq 0, y \geq 0$

Step 3: Formulate the objective functions

- Objective is Maximise Contribution. So objective is:
- Maximise $15x + 25y$

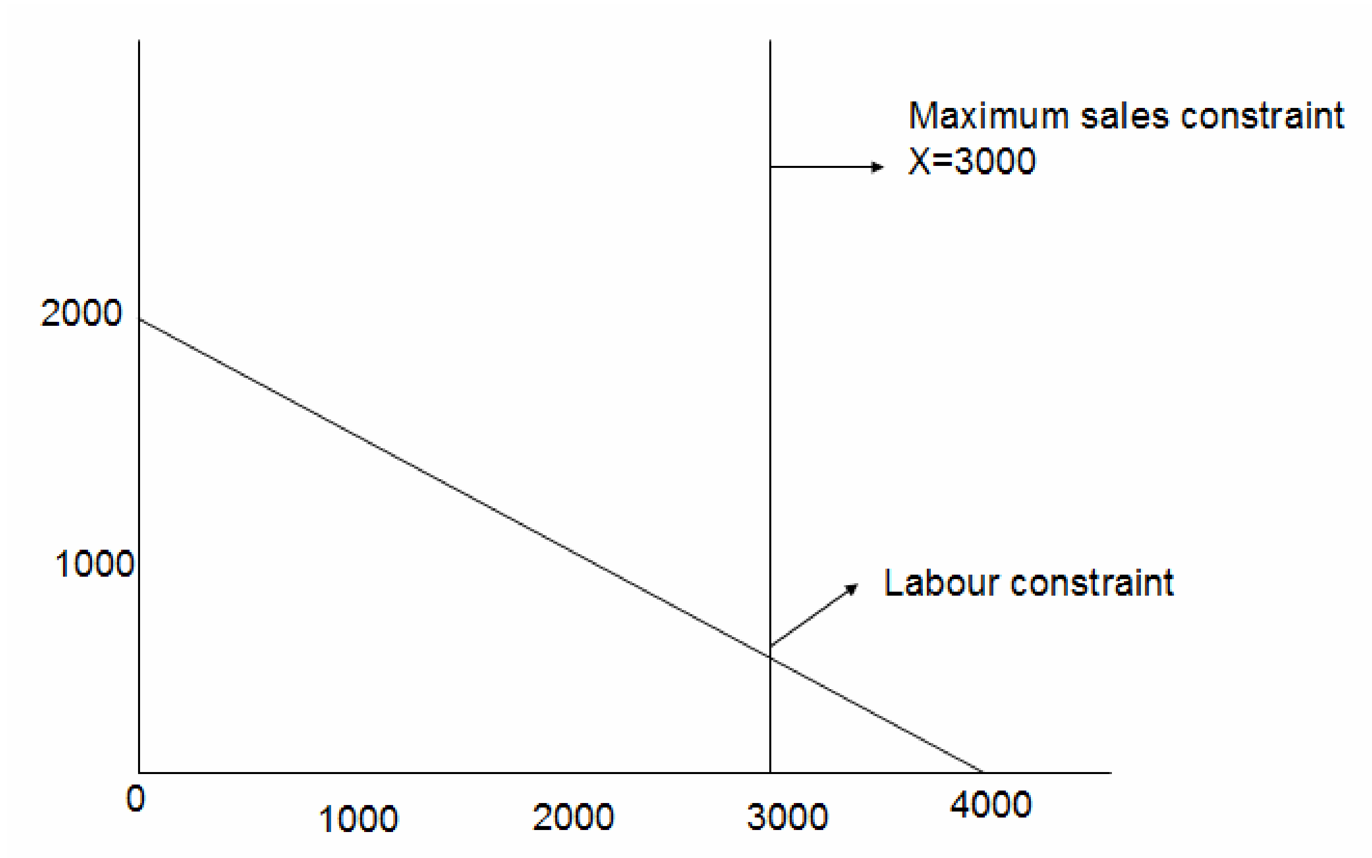
Step 4: Graph the constraints

- $2.5x + 5y \leq 10000$

  $2.5x + 5y = 10000$
- Put $x = 0$, then $y = 2000$

  $Y = 2000$
- If $y = 0$, $x = 2.5x + 0 = 10000$, $X = 4000$
- Draw the two points (0, 2000) and (4000, 0) on a graph to represent the line for labour constraint

Step 4: Graph the constraints



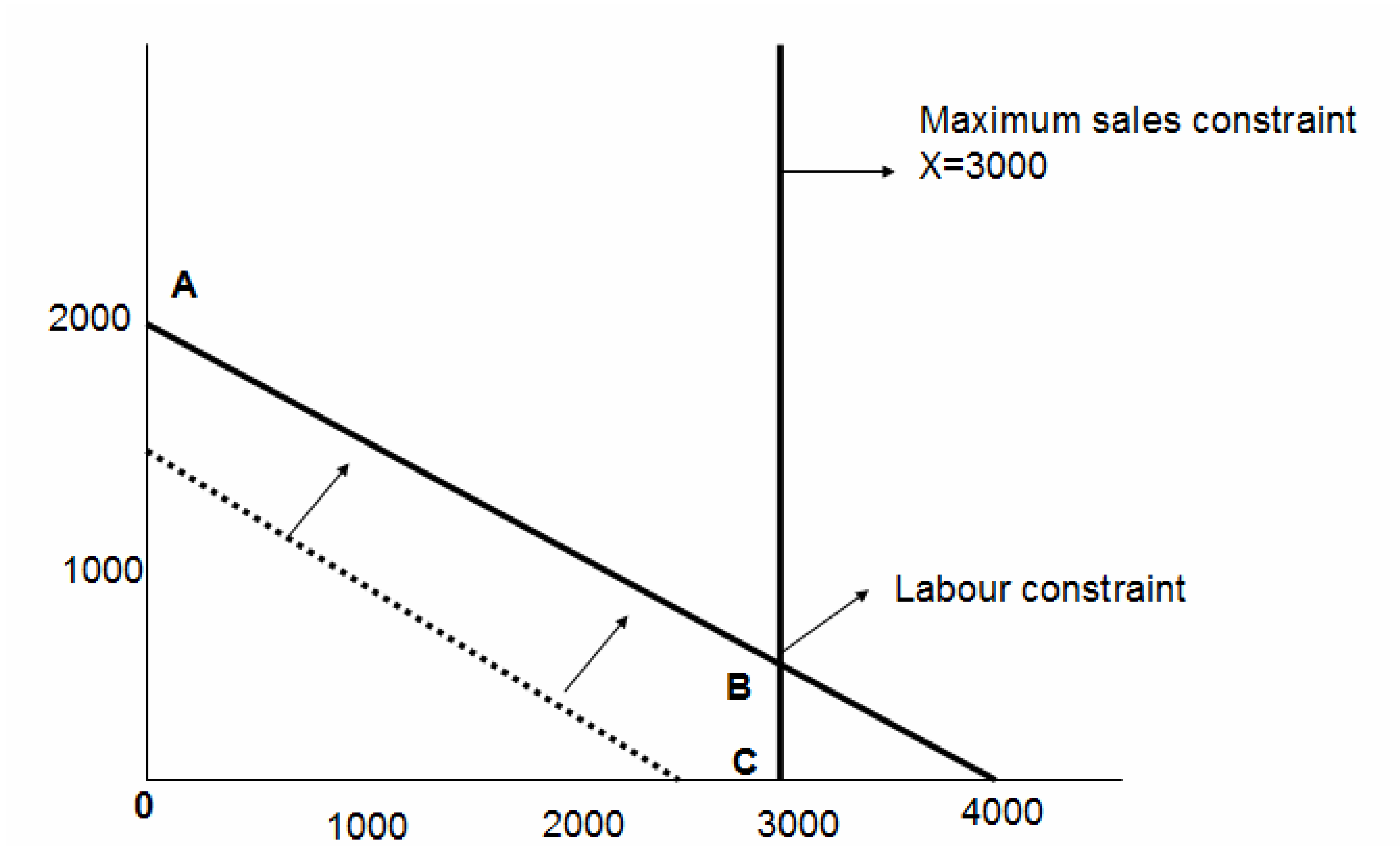
Y units

X units

Step 5: Graph objective function

- Take any reasonable value of maximum contribution
- 15x + 25y = 37500
- If x = 0, y = 1500 and If y = 0, x= 2500
- To draw 15x + 25y =37,500 draw (0, 1500) and (2500,0) as the two points.
- *NOTE: Any contribution line can be drawn. The purpose is only to establish the slope*

Step 5: Graph objective function



Step 6: Determine the optimal solution

- **Method 1:** Calculate the contribution earned at each point A, B and C.
  - Point A =£15 * 0 + £25 * 2000=£50000
  - Point B = £15 * 3000 +£25 *500 =£57,500
  - Point C = £15 * 3000 = $45000
  - Pont B gives maximum contribution

Step 6: Determine the optimal solution

- **Method 2:** Slide the objective function line away from the origin to the furthest possible point
  - The optimal solution is B where sales constraint intersects labour constraint

Algebraic Solutions

- Optimal solutions can be arrived at using simultaneous equations instead of graphs.

Example:

- John manufactures garden chairs and armchairs. Each product passes through a cutting process and an assembling process. One garden chair makes a contribution of $50 and takes 6 hours cutting time and 4 hours assembly time. One armchair makes a contribution of $40 and takes 3 hours cutting time and 8 hours assembly time There is a maximum of 36 cutting hours available each week and 48 assembly hours
- Find the output that maximises contribution.

Solution

- Let x = number of garden chairs produced each week
- Y= number of armchairsproduced each week
- Constraints are:
- 6x +3y =36
- 4x +8y =48 with x ≥0, y ≥0
- Solving for x and y gives, x = 4 and y= 4
- Thus maximum contribution = 4 x $50 + 4 X$40= $360

Limitations of Linear Programming

- Linear relationships must exist
- Only suitable when there is only one objective function
- It is assumed that variables are completely divisible
- Assumed that situation remains static in all respects